FLATNESS OF NONLOCAL PHASE TRANSITION IN LOW DIMENSIONS

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Abstract. We consider nonlocal functionals, like fractional perimeter or the energy functional associated to the fractional Allen-Cahn equation, in dimensions $n = 2$ and $n = 3$. It is known that nonlocal minimal surfaces are flat in dimension 2, and that the level sets of minimizers for the s-fractional Allen-Cahn equation in the all space are flat in dimension 2 for any s and in dimension 3 for any $1/2 \leq s < 1$. We give a quantitative version of these results, in the following sense: we prove that the level sets of minimizers in a ball of radius R are nearly flat in B_1, when R is large enough. More precisely, we establish a quantitative estimate on how "close" these surfaces are (in the L^1-sense and the in the L^∞-sense) to be a plane, depending on R. Our approach does not use the Caffarelli -Silvestre extension, and can be applied to more general nonlocal functional, like, for example, the anisotropic fractional perimeter. This is a joint work with Joaquim Serra and Enrico Valdinoci.